Multiple scattering effects under vertical sounding of equatorial ionosphere

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Abstract

This paper presents a theoretical study of the influence of electron density small-scale irregularities on radio propagation under vertical sounding of the equatorial ionosphere. The sounder is assumed to radiate in the plane perpendicular to geomagnetic field lines. An approximate analytical solution of the equation of radiation energy balance in a plane layer of randomly inhomogeneous plasma has been obtained for this case. Analysis of the results shows that multiple scattering leads to attenuation of signal power and change of the signal arrival angles in the sounder vicinity.

1. Introduction

As it is well-known, radio wave propagation in the ionosphere can be affected by electron density irregularities. In particular, scattering from small-scale random irregularities can lead to spatio-angular redistribution of radiation reflected from the ionospheric plasma layer. It was found recently that optical thickness of the ionosphere for the scattering process for HF radio waves is considerably greater than unity (Bronin et al., 1993). Therefore, a correct analysis of wave propagation in this case requires accounting for the multiple scattering effects.

The radiation energy balance (REB) equation describing radiation transfer in an optically thick plane layer of randomly inhomogeneous plasma for the case of total reflection has been obtained in Zabotin et al. (1998a). As shown in Bronin and Zabotin (1993), the REB equation is a particular case of a more general radiation transfer equation obtained in Bronin and Zabotin (1992) and can be deduced from the latter by means of transition to the invariant ray coordinates. The details of this transition are briefly described in the Appendix. Invariant ray coordinates allow one to readily take into account the refraction of waves and to represent the equation in its simplest form. They also allow one to introduce the small-angle scattering in the invariant ray coordinates (SASIRC) approximation and to suggest an analytical solution of the REB equation in this approximation (Zabotin et al., 1998a). Few effects that require coherent summation of wave amplitudes for their description (for example, the enhanced backscattering phenomenon) cannot be considered within the transfer theory framework. But these are relatively insignificant in comparison with multiple scattering effects for radio wave propagation in the ionosphere. The REB equation and its solution in the SASIRC approximation are new objects of the scattering theory, therefore, comprehensive investigation of their properties is of great interest.

A recent paper considers a model problem of total reflection of the HF radiation emitted by a ground based point source with a plane directivity diagram.
from the equatorial ionosphere F region containing random irregularities of electron density. The directivity diagram plane orientation is supposed to be orthogonal to the geomagnetic field and to the irregularities infinitely stretched along the magnetic field lines. That is why all scattering processes occur within this single plane. The REB equation solution in the SASIRC approximation has a relatively simple form in this case and its investigation can be carried out by simple means.

2. Problem formulation and initial assumptions

Let us introduce the Cartesian system of coordinates with ground based point source in its origin $O$. The $z$-axis is directed vertically and the $xOy$-plane represents the Earth’s surface. Let the ionospheric plasma layer be situated at distance $h_0$ from the ground and it has a linear dependence of an average electron concentration on height $(dz/dv=H$, where $v=\omega_0^2/\omega^2$, $\omega_0$ is the plasma frequency, $\omega=2\pi f$, $f$ is frequency of the wave). The source radiates only in the $xOz$-plane and its directivity diagram is wide and symmetric with relation to the $z$-axis. At the equatorial latitudes the geomagnetic field is nearly horizontal, therefore, we shall assume that the magnetic field vector $H$ is perpendicular to the $z$-axis and it is directed along the $y$-axis (see Fig. 1).

It is accepted that small-scale irregularities of the ionospheric F region are strongly stretched along the geomagnetic field (Fejer and Kelley, 1980). In our theoretical investigation we will use the approximation of infinitely stretched irregularities, with the spectrum of the following form

$$F(k) = C_A(1 + \kappa_2^2/\kappa_0^2)^{-\nu/2}\delta(k),$$

where $\kappa_2$ and $\kappa_0$ are, respectively the vector $k$ (irregularity spatial harmonic) components orthogonal and parallel to the magnetic field lines, $\kappa_{10}=2\pi/l_{0\perp}$, $l_{0\perp}$ is the external scale of the irregularity spectrum, $\nu$ is the spectrum index, $\delta(x)$ is the Dirac delta-function,

$$C_A = \delta_R^2 \frac{\Gamma(\nu/2)}{2\pi l_{0\perp}} \left[ \int \left( \frac{\nu-2}{2} \right)^{1/2} K_\nu - \frac{2}{2} \right]^{-1}$$

is a normalizing constant, $\Gamma(x)$ is the gamma-function, and $K_\nu(z)$ is the Macdonald function (Abramowitz and Stegun, 1972). The $\delta_R$ quantity characterizes the level of irregularities $\Delta N/N$. In the mathematical theory of random fields it corresponds to the structure function of the irregularity field for the scale length $R$ (Rytov et al., 1989). The expression (2) for $C_A$ is obtained using the condition of equality of the structure function in the scale $R$ to $\delta_R^2$. It is assumed that the scale length, $R$, is determined in the transversal direction to the magnetic field.

As is known, the scattering condition of a wave with the wave vector $k_i$ on the irregularity harmonic $k$ is $k = k_i - k_s$, where $k_i$ is the wave vector of a scattered wave. Therefore, under chosen relative orientation of a directivity diagram and magnetic field (irregularities)
the scattered wave vector $\mathbf{k}$, always lies in the $xOz$-plane. The radiation never leaves this plane. So, the problem in its present formulation is two-dimensional. We will be interested in both power redistribution and arrival angles change of reflected radiation on the ground that it is along the line coinciding with the $x$-axis of the coordinate system.

Collisional absorption will not be taken into account in our model. In reality it corresponds, for example, to the nighttime radio reflections from the ionospheric $F$ layer. We will also use an isotropic plasma approximation for the wave refractive index. As our investigation undertaken in other cognate work (Zabotin et al., 1998b) shows, taking into account the magnetic field influence on the wave refractive properties does not change the results significantly.

3. Basic relations

The parameters determining (without random irregularities) the shape and location of a ray path in a regular plane plasma layer are called invariant ray coordinates. In the considered problem the invariant ray coordinates are polar and azimuth angles $\theta$ and $\varphi$ of the ray incidence onto the layer and the point $\mathbf{r} = (x, y)$ of its return onto the ground ($xOy$-plane). The radiation energy balance equation describing radiative transfer in a plane layer of randomly inhomogeneous plasma has the form (Bronin and Zabotin, 1993; Zabotin et al., 1998a)

$$\frac{d}{dz} P(z, \mathbf{r}, \theta, \varphi) = \int \left[ Q(z, \theta, \varphi, \theta', \varphi') P(z, \mathbf{r} - \hat{Q}(z, \theta, \varphi, \theta', \varphi')) - P(z, \mathbf{r}, \theta, \varphi) \right] d\theta' d\varphi',$$

where $P(z, \mathbf{r}, \theta, \varphi)$ is the energy flux on the ground surface, $Q(z, \theta, \varphi, \theta', \varphi') = \sigma(\theta, \varphi, \theta', \varphi') c^{-1}(z, \theta, \varphi) \sin \theta'$ is a differential scattering cross section, $c(z, \theta, \varphi)$ is the cosine of the inclination angle of the ray path with relation to the $z$-axis, corresponding to the invariant angles $\theta, \varphi'$; $|d\Omega_c/d\Omega|^{-1}$ is the Jacobian of the coordinate transformation from the current polar and azimuth angles of the wave vector to the invariant ray variables $\theta', \varphi'$; the vector function $\hat{Q}(z, \theta, \varphi, \theta', \varphi')$ represents the displacement of the arrival point at the ground surface of a ray which has angular coordinates $\theta', \varphi'$ after scattering at the $z$-level relative to the arrival point of an incident ray with angular coordinates $\theta, \varphi$. The scattering cross section for isotropic plasma is

$$\sigma = \frac{1}{2} \sigma_0 \kappa^2 F(\mathbf{k}),$$

where $\sigma_0 = 2\pi/c$, $F(\mathbf{k})$ is the spatial spectrum of the irregularities (Rytov et al., 1989).

The SASIRC approximation is valid if the most probable difference between angles $\theta', \varphi'$ and $\theta, \varphi$ under each scattering act is small. This situation is typical for irregularity spectra in which irregularities with scales larger than the sounding wavelength dominate. The heuristic basis for this approximation ensues from analysis of the Poeverlein construction (Zabotin et al., 1998a). The basic term of the solution of the REB equation in the SASIRC approximation has the form (formula (15) in Zabotin et al., 1998a)

$$P_1(z, \mathbf{r}, \theta, \varphi) = P_0 [\mathbf{r} + \mathbf{D}(z, \theta, \varphi)],$$

where $P_0(\mathbf{r}, \theta, \varphi)$ is the radiation energy flux on the ground in the absence of irregularities,

$$\mathbf{D}(z, \theta, \varphi) = \int_0^z dz' \int d\theta' d\varphi' Q(z', \theta, \varphi, \theta', \varphi') \Phi(z', \theta, \varphi, \theta', \varphi'),$$

where integration over $z'$ is carried out along the ray trajectory. The undisturbed distribution $P_0$ is determined by antenna pattern of the sounder and regular properties of the plasma layer. The calculation of $P_0$ can be carried out on the basis of the ray approximation of the geometrical optics method. Such calculations have been made in a series of publications (see, for example, Budden, 1985; Kravtsov and Orlov, 1990). As is clear from (6), the transformation of radiation distribution caused by multiple scattering consists of the displacement $\mathbf{D}(z, \theta, \varphi)$ of the spatial variable $\mathbf{r}$ without changing the function $P_0$ form. The latter effect can be qualitatively characterized as the deformation of the radiation field.

4. Arrival angles and intensity

In this section the above general theory is employed for the calculation of reflected radiation flux distribution for the problem stated in the second section. As the problem is two-dimensional, one can write merely variable $x$ instead of $\mathbf{r}$ supposing that $y = 0$. In the absence of irregularities spatio–angular distribution of the reflected radiation flux on the ground surface ($z = 0$) can be represented in the form
\[
P_0(x, \theta, \varphi) = \tilde{P}_0(x) \delta(\cos \theta - \cos \varphi_0(x)) \]
\[
\left[ \delta(\varphi) + \delta(\varphi - \pi) \right],
\]
where \(\tilde{P}_0(x)\) and \(\varphi_0(x)\) are energy flux and ray arrival angles in the point \(x\) in the absence of irregularities. Substituting (8) into (6) and integrating over the angles one can obtain the following expression for the energy flux at point \(x\)
\[
\tilde{P}_1(x) = \tilde{P}_0(x + D_x(0^*, \varphi)) \left| 1 + \frac{\partial \varphi_0(x + D_x(0^*, \varphi))}{\partial x} \frac{\partial D_x(0^*, \varphi)}{\partial \theta} \right|^{-1}.
\]

Here \(D_x(\theta, \varphi)\) denotes the \(x\)-component of the vector \(D(0, 0, \theta, \varphi, 0^*)\) is the new ray arrival angle which is determined by the equation
\[
x_0(0^*, \varphi) - x = D_x(0^*, \varphi),
\]
where \(x_0(\theta, \varphi)\) is the arrival point of the ray with the invariant angles \(\theta\) and \(\varphi\) in the absence of irregularities. In formulae (9) and (10) one should substitute \(\varphi = 0\) when \(x \geq 0\) and \(\varphi = \pi\) when \(x < 0\). Due to the problem symmetry function \(\tilde{P}(x)\) is even: \(\tilde{P}(-x) = \tilde{P}(x)\). Therefore, it is reasonable to consider \(\tilde{P}(x)\) only under \(x \geq 0\) (when \(\varphi = 0\)). A common expression for function \(D_x(\theta, \varphi)\) for a layer of isotropic plasma with linear profile of electron concentration can be found in Zabotin et al. (1998a) [formula (31)]. In that expression the polar and azimuth angles \(\gamma\) and \(\beta\) of the wave vector in the ‘magnetic’ coordinate system (which the \(Oz\)-axis is parallel to the magnetic field and the \(Ox\)-axis coincides with the \(Ox\)-axis of our coordinate system) are used. These angles are related to the invariant ray angles. For \(\varphi = 0\) and magnetic inclination \(\gamma = 90^\circ\) this relation has the form
\[
x_0(\theta, \varphi) = x_0(\theta, \varphi) = 0
\]
\[
\cos \beta_1, = \frac{\sin \theta}{\sqrt{1 - v}},
\]
where subscript 1 and the upper sign are for the ascending branch of a ray trajectory, and subscript 2 and the lower sign are for the descending branch of a ray trajectory. Substituting \(\varphi = 0\), \(\gamma = 90^\circ\) and expressions (11) into the expression for \(D_x\) one can obtain
\[
D_x(\theta, \varphi, 0) = \frac{\pi k^3 H^2 C_A}{2 k_0} \int_0^\cos^2 \theta d\theta
\]
\[
\left[ 2(\sqrt{1 - v} + (1 - v) \sin^2 \beta - \sin \beta \sqrt{1 - v}) + \frac{h_0}{H}(1 - v) \sin \beta \right]^{-1/2} - 2 \sin \theta \cos \theta
\]
\[
\left( 1 + 4 k_0^2 v_0^2 (1 - v) \sin^2 \left( \frac{\beta - \beta_1}{2} \right) \right)^{-1/2}
\]
\[
+ \text{similar term in which } \beta_1 \text{ is replaced by } \beta_2
\]
\[
+ \text{before expression } \sqrt{\cos^2 \theta - \gamma} \text{ is replaced by } -
\]

In a linear layer of isotropic plasma dependence of the arrival point \(x_0\) upon the arrival angle \(\theta\) when \(\varphi = 0\), in the absence of irregularities has the form (Budden, 1985)
\[
x_0(\theta) = x_0(\theta, \varphi = 0)
\]
\[
= \frac{k_0}{H} \cos \theta + 4H \sin \theta \cos \theta.
\]

5. Calculation results and their analysis

Because of the complexity of the expression (12) for
function $D_x(\theta)$ Eq. (10) for the modified arrival angle $\theta^*$ was solved numerically using the dichotomy method (Dennis and Shnabel, 1983). The calculations were carried out for the following set of parameters: $h_0=150$ km; $H=100$ km; $v=2.25, 2.5, 2.75$; $l_{\perp}=10$ km; $R=1$ km; $f=5$ MHz. The irregularity parameters used may be considered typical for the quiet equatorial conditions (Szuszczewicz, 1987; Fejer and Kelley, 1980).

Function $D_x(\theta)$ behavior for small angles is shown in Figs. 2 and 3. It has a minimum at $\theta \approx 0.5^\circ$, which becomes deeper when irregularity level $\delta_R$ or the spectrum index $v$ increases. At larger angles $D_x$ grows monotonically with $\theta$ achieving zero under $\theta=90^\circ$. It makes sense to analyze Eq. (10) finding the intersection of the functions $x_0(\theta)$ and $D_x(\theta)$ graphically. The root of the equation is abscissa of the intersection point. One example of the solution is shown in Fig. 4. From this graph it is clear that the modified arrival angle $\theta^*$ is always smaller than arrival angle $\theta_0$ in the absence of irregularities.

Fig. 5 displays dependence of the arrival angle $\theta^*$ on $x$ for different irregularity levels. Undisturbed angle behavior $\theta_0(x)$ is also shown in the graph (by a dashed line) for comparison. As it is seen from Fig. 5, because of multiple scattering there appears the region in the sounder vicinity with rays arriving nearly vertically ($\theta^* \approx 0^\circ$). These rays ‘huddle’ to the vertical direction and became nearly parallel to each other. This spatial range grows with the irregularity level increase. This tendency is also kept for higher irregularity levels ($\delta_R > 0.004$).

The relative alteration of signal power $\bar{P}_i/\bar{P}_0$ is shown in Fig. 6 as a function of the distance $x$ from the sounder for different irregularity levels. This graph demonstrates a common feature of the multiple scattering effects under vertical sounding of the ionosphere (Zabotin et al., 1998a): considerable attenuation of the reflected signal power in the sounder vicinity. The attenuation increases together with the irregularity level. The spatial range of the attenuation region also increases with $\delta_R$. Comparing Figs. 5 and 6 one can...
see that the attenuation region coincides with the region in which arriving rays are nearly vertical (parallel).

It should be noted that Eq. (10) sometimes formally has several roots. At some sufficiently high values of the irregularity level \( \delta_R \) and the spectrum index \( \nu \) there appears a possibility of triple intersection of the curves \( x_0(\theta) \sim x \) and \( D_x(\theta) \). Such an example is shown in Fig. 7. In this case there is a finite interval of \( x \) in which Eq. (10) has three roots. The dependence of \( D_x(\theta) \) upon \( \delta_R \) and \( \nu \) entails an increase of this spatial region range with \( \delta_R \) and \( \nu \). We do not consider this root multiplication physically meaningful. Perhaps, in the region where it occurs the approximations used under obtaining solutions (9) and (10) are too rough. As is clear from Fig. 7, in the sounder vicinity \( (x \approx 0) \) and at the large distances there always exists the only root corresponding to a single arriving ray.

Our consideration can be illustrated by the experimental data obtained using dyanosonde at Huancayo (magnetic-equatorial region), which, in our view, demonstrates the anomalous attenuation effect. Fig. 8, taken from the work of Wright et al. (1996), compares data from four superimposed Huancayo daytime and early evening absolutely unspread recordings [Part (a)], with four superimposed nighttime spread F recordings in Part (b). In each part, the ionograms and echo intensities (dB) are shown on independent vertical scales versus radio frequency. The purpose of combining multiple recordings in this way was to obtain significant intensity patterns within periods of reasonably stationary conditions, in spite of short-period fading and focusing effects. All intensities were compensated for range dilation, to a reference height of 100 km. Note that in Part (b) echo amplitudes for frequencies exceeding 6 MHz have been lowered by 15 to 20 dB relative to their undisturbed mean value (marked by a reference line). Irregular structure caused additional attenuation of the signal and the only possible explanation of this phenomenon at present may be connected with multiple scattering effects (namely, with the attenuation of the reflected signal power in the sounder vicinity, see Fig. 6) stated in this paper.

6. Conclusion

The subject of analysis in this paper is multiple scattering effects that may take place under vertical sounding of the magnetic-equatorial ionosphere. The model assumption about the plane directivity diagram of the radio sounder has allowed us to treat the problem by relatively simple means. It has been found that anomalous attenuation of the sounding signal in the equatorial conditions may achieve several tens of decibels. There is no suppression of this effect in comparison with its mid-latitude manifestations.

Another interesting feature of the vertical sounding signal arising in the multiple scattering regime is ‘parallelization’ of the reflected radiation flux in the sounder vicinity (in the even region where anomalous attenuation takes place). An optically thick ionospheric layer acts like a parabolic mirror with the radio sounder in its focus. The possibility of direct observation of the latter effect is not clear as yet. But it may appear
Fig. 8. Comparison of unspread ionograms [Part (a), top] with fully-developed equatorial spread F [Part (b), bottom]. Each part displays echo stationary-phase group range (km) and echo intensity (dB). Four Huancayo dyansonde ionograms, 2040 to 2140 UT, 83-03-14 (O-mode echoes only), are superimposed in each part: in (a), at 1640, 1635 LT (pre-sunset, with foE and Es-q near 100 km); and 1900, 1905 LT (post-sunset, after photochemical decay of D and E regions); in (b), at 2130 to 2140 LT, 83-03-14. A reference level of 80 dB is marked in each panel (from Wright et al., 1996).
essential for energetics of the secondary reflection from the ionosphere.

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Appendix

The equation of radiation transfer in a randomly irregular plasma can be written in the form (Bronin and Zabotin, 1993)

\[
\frac{n^2}{\cos \theta_g} \frac{d}{ds} \left( R(z, t, z, \beta) \cos \theta_g \right) = \int \left[ \sigma(x, \beta, z, x', \beta') R(z, t, \beta) \right] d\Omega_g,
\]

(14)

where \( z, t = (x, y) \) are vertical coordinate and horizontal radius vectors of a point, \( R(z, t, z, \beta) \) is the radiation energy flux per unit solid angle \( d\Omega = \sin z \ dz \ d\beta \) in the direction determined by angles \( z, \beta \) of the wave vector \( k \), \( n \) is the refractive index, \( \theta_g \) is the angle between the wave vector and the group velocity vector, \( d/ds \) is the derivative along the ray trajectory. In (14) it is assumed that radiation is monochromatic, medium is stationary and conversion of normal waves is negligible.

The derivation of the radiation energy balance Eq. (3) in plane plasma layer from Eq. (14) can be done in the following way.

At the first step one should transit from the ray intensity \( I(z, t, z, \beta) \) determined along the wave vector to the ray intensity \( I(z, t, \beta, \phi) \) which is determined as the energy flux in the direction of the group velocity vector with angle coordinates \( \beta, \phi \) and per unit solid angle \( d\Omega_g = \sin \theta \ d\theta \ d\phi \).

The second step consists of the introduction of the novel function \( P \equiv I \mid \mathcal{K}_g \), where \( \mathcal{K}_g \) is the Gauss curvature of the refractive index surface. It should be noted that function \( P \) has a sense of the ray intensity only out of the plasma layer, where \( \mathcal{K}_g = 1 \).

At the last step one should realize the transition to the invariant ray coordinates \( \mathbf{p}, \theta, \phi \). To obtain the REB equation in its final form (3) one should use the following relations:

\[
\frac{d}{dz} P(z, \mathbf{p}, \theta, \phi) = C(z, \theta, \phi) \frac{d}{dz} P(z, \mathbf{p}, \theta, \phi),
\]

\[
\frac{d\Omega_g}{d\Omega} = \cos \theta_g \cos \phi \ \frac{n^2}{C(z, \theta, \phi)}
\]

\[\sigma(\theta, \phi; \theta', \phi') n^2 \cos \theta_g' = \sigma(\theta', \phi'; \theta, \phi) n^2 \cos \theta_g,\]

where \( C(z, \theta, \phi) = \cos \theta \).

References